83. *The Interchange of Heat between a Gas Stream and Solid Granules. Part I.*

By J. R. Arthur and J. W. Linnett.

The temperature distribution in a bed of charcoal granules through which air is streaming and to which heat is being supplied by a heater embedded in the charcoal has been studied. The plot of the logarithm of the temperature at the steady state against the distance down the column gives two straight lines, one for the region on the near side of the heater and the second for the region on the far side. The slope of the log T–x graph is dependent on the flow rate but independent of the rate at which heat is supplied by the heater. A mathematical treatment is described which accounts precisely for the results obtained. According to this treatment the rate of interchange of heat between the gas and the solid is proportional to the temperature difference between them. Moreover, the rate of interchange of heat is rapid compared with the conduction of heat through the charcoal, so it is concluded that, at the steady state, the temperature of the air follows closely the charcoal temperature at all points in the system examined.

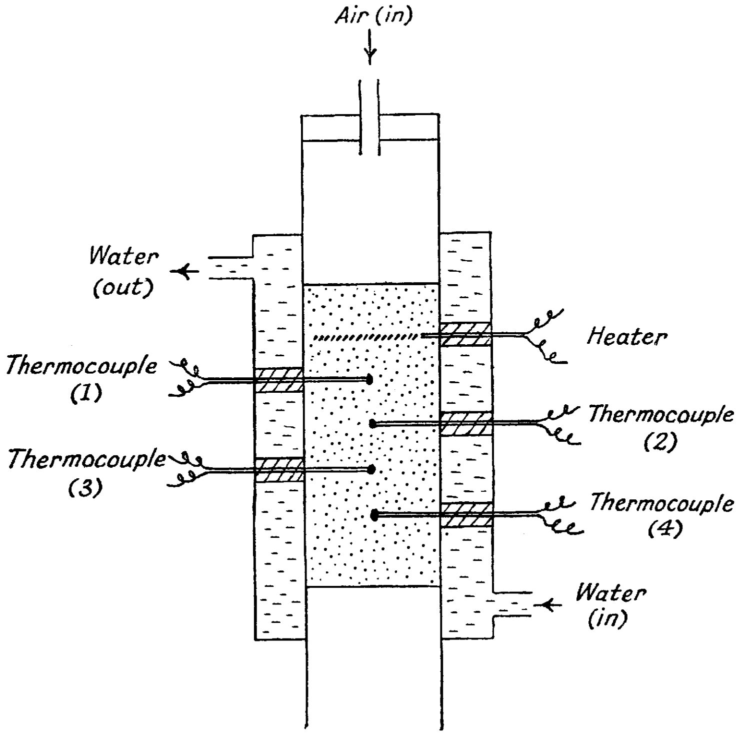
Gilbert (*Engineer*, 1930, **150**, 500, 530, 574, 617, 640) and Furnas (*Ind. Eng. Chem.*, 1930, **22**, 26, 721) have studied the passage of heat between a gas stream and a bed of granules maintained at different temperatures. We studied the passage of heat between an air stream and a bed of charcoal granules in a system which was constructed to resemble a catalytic system. In the latter, heat is produced by a chemical reaction over a volume of the granular bed, while in our idealised arrangement heat was produced electically at a given layer. Results obtained for a real catalytic system will be published later.

EXPERIMENTAL.

The charcoal granules used to make the granular bed were graded between 8 and 18 B.S.S. and were dried by heating to 120°. The air used was dried with calcium chloride and concentrated sulphuric acid. The granular bed was contained in a metal tube, 29·7 cm. long and of 4·7 cm. internal diameter. The tube was set vertically and water-jacketed so that the wall temperature remained constant. The tube was fitted with short side tubes at intervals of 1·5 cm. down its length through which passed the leads to the heating element and the thermocouples that measured the temperature of the granules (see Fig. 1).

The heating element was a wire grid on a square glass frame of side 2·5 cm. The wire was 33 S.W.G. constantan. Its resistance was about 3 ohms. The current, supplied by two 2-volt accumulators, was

FIG. 1.



kept constant with a variable resistance and measured in arbitrary units on an ammeter. The heating element was placed perpendicular to the axis of the tube, and the leads passed out through a rubber stopper in a side tube. The inner part of the stopper was flush with the tube wall. Copper–constantan thermocouples were used. In a preliminary experiment the effect of the diameter of the wires forming the thermocouple was investigated. The temperatures recorded by three thermocouples at a constant bed temperature are shown in Table I. The recorded temperature varies approximately linearly with the area of the wires. Since the area of the two 40 S.W.G. wires is very small, it may be presumed that a thermocouple made from these records fairly well the temperature of the charcoal at the junction.

TABLE I.

Wires used in thermo-couple.	100 × total area of two wires, sq. mm.	Temp. recorded by thermocouple.
28 S.W.G. Constantan 24 S.W.G. Copper	27·16	33·3°
33 S.W.G. Constantan 32 S.W.G. Copper	11·06	40·1
40 S.W.G. Constantan 40 S.W.G. Copper	2·26	44·8

The error is probably less than 1°. The junction between the two wires was made in a sphere of solder about the size of a charcoal granule. The two wires were led in through fine glass tubes which passed through a rubber stopper. The cold junctions were kept at 0°. In a single experiment four thermocouples were used. These were calibrated by placing their hot junctions in steam, all giving $0{\cdot}4286 \pm 0{\cdot}0003 \times 10^{-2}$ volt. The E.M.F.'s were measured on a Tinsley general utility potentiometer standardised against a Weston cadmium cell. It is presumed in what follows that the thermocouples measure the charcoal temperature since the sphere of solder at the junction resembled a granule of charcoal. The air flow rate through the bed, measured on a capillary flowmeter, could be varied between 2000 and 5000 c.c./min.

(1) In the first set of experiments four thermo-junctions were located along the axis of the charcoal column on the side of the heating element remote from the air inlet. The charcoal column was 11·3 cm. long, and the heating element 1·05 cm. from the top. The junctions were placed 1·40, 2·65, 4·50 and 5·85 cm. from the element. The E.M.F. of each junction was read every two minutes. When the charcoal temperature was steady and uniform the air was turned on and the final steady temperature of each junction recorded. This was carried out for flow rates (F) of 2000, 2500, 3000, 3500, 4000, 4500, and 5000 c.c./min. The following tables give only the final steady temperature, and the mathematical treatment is of the steady state and not of the approach to that state.

Table II gives the differences between the steady temperatures recorded by the thermocouples and the wall temperature (which was $13\frac{1}{4}$°) for an experiment carried out with a flow rate of 3500 c.c./min.

TABLE II.

Distance of thermocouple from heater, cm. ...	1·40	2·65	4·50	5·85
Final steady temp.—wall temp.	28·50°	18·98°	10·37°	6·63°
$\log_{10} T$	1·4549	1·2783	1·0156	0·8215

TABLE III.

Thermocouple at (cm.).	log T for flow rates given in c.c./min. 2000.	2500.	3000.	3500.	4000.	4500.	5000.
1·40	1·4427	1·4549	1·4517	1·4549	1·4448	1·4293	1·4244
2·65	1·2007	1·2386	1·2547	1·2783	1·2852	1·2825	1·2873
4·50	0·8424	0·9160	0·9637	1·0156	1·0431	1·0548	1·0800
5·85	0·5777	0·6761	0·7466	0·8215	0·8665	0·8932	0·9160
Slope of line	0·1948	0·1768	0·1584	0·1444	0·1314	0·1218	0·1128

The temperature difference is represented by T. Table III gives the values of $\log_{10}T$ for the seven flow rates. If $\log_{10}T$ is plotted against x, the distance from the heater, a straight line is obtained, the slope of the line increasing with decreasing flow rate. The lines for $F = 2000$ and $F = 5000$ c.c./min. are shown in Fig. 2.

Further experiments were carried out at $F = 4000$ and $F = 5000$ c.c./min. to see whether this important linear relationship held for smaller values of x. The thermocouples were on the axis 0·20, 1·20, 1·45 and 4·55 cm. from the heater. Table IV gives the experimentally determined values of log T for the two flow rates. The results for $F = 5000$ c.c./min. are included in Fig. 2.

TABLE IV.

Thermocouple at (cm.).	log T for flow rates given in c.c./min. 4000.	5000.
0·20	1·6077	1·5514
1·20	1·4881	1·4564
1·45	1·4293	1·4147
4·55	0·9925	1·0619

The slopes of the $\log_{10}T$–x lines are given in the last row of Table III. It is found empirically that the reciprocal of the slope varies linearly with the flow rate.

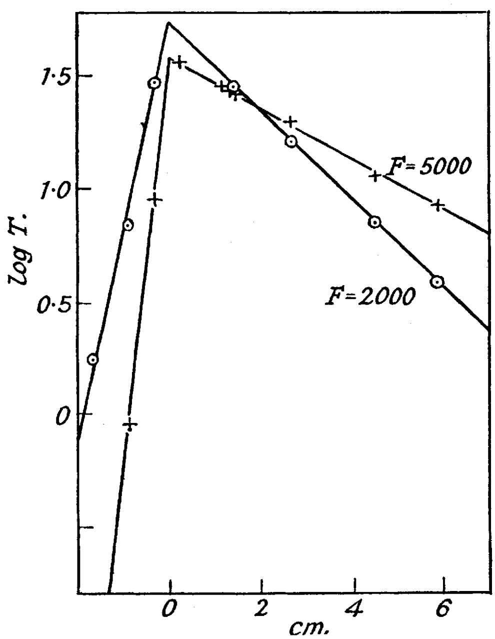
(2) A second series of measurements was made with thermocouples on the axis of the tube on the side of the heater adjacent to the incoming air. They were 0·30, 0·90, 1·70, and 2·85 cm. from the heater, but the junction at 2·85 cm. never showed any appreciable heating. Except at the lowest flow rate the temperature at 1·70 cm. from the heater was raised less than 1° so the percentage error in this reading was quite high. The values of $\log_{10}T$ at the steady state are shown in Table V for flow rates between 2000 and 5000 c.c./min. If the temperature at $x = 0$, obtained by extrapolating the results listed in Table III for temperatures beyond the heater, is used in conjunction with the figures in Table V it is found that,

TABLE V.

Thermocouple at (cm.).	log T for flow rates given in c.c./min. 2000.	2500.	3000.	3500.	4000.	4500.	5000.
0·30	1·4658	1·3988	1·3413	1·2564	1·1917	1·1264	0·9503
0·90	0·8336	0·6945	0·5696	0·3976	0·2925	0·1164	−0·0407
1·70	0·2433	−0·0407	−0·1694	−0·4277	−0·4557	−0·7287	−0·4857
Slope of line	0·950	1·101	1·202	1·387	1·474	1·620	1·718

for negative values of x also, log T varies linearly with x. Fig. 2 shows the graphs for $F = 2000$ and 5000 c.c./min. The slopes of the lines are given in Table V. It is found empirically that the slope varies linearly with the flow rate.

FIG. 2.



(3) In a third set of measurements the effect of varying the current through the heater was examined. Measurements were made with currents of 20, 24, 28, and 30 units, 28 units having been used in the first and second series. The thermocouples were placed as in the first series and a flow rate of 4000

FIG. 3.

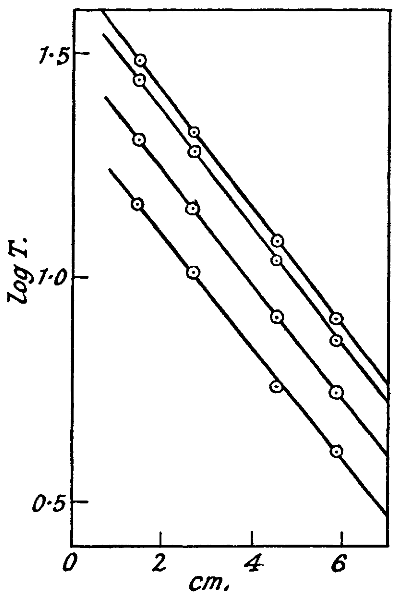
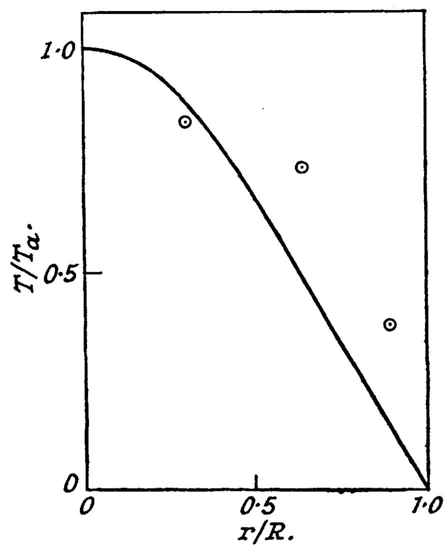


FIG. 4.



c.c./min. was used. The results are listed in Table VI and shown graphically in Fig. 3. It will be seen that the slope of the $\log_{10}T$–x plot is independent of the rate at which heat is produced by the element, being dependent, for a given bed, only on the air-flow rate.

TABLE VI.

Thermocouple at (cm.).	log T for different heating currents ($F = 4000$). Units: 20.	24.	28.	30.
1·40	1·1648	1·3117	1·4448	1·4871
2·65	1·0146	1·1578	1·2852	1·3287
4·50	0·7661	0·9196	1·0431	1·0850
5·85	0·6162	0·7484	0·8665	0·9123

(4) In a fourth series of measurements the temperature distribution across the column at a given depth (4·3 cm. beyond the heater) was examined. Thermocouples were placed 0·0, 0·7, 1·5, and 2·1 cm. from the axis, and flow rates of 2000, 3000, 4000, and 5000 c.c./min. were used. It was found that T/T_a (T_a is the temperature on the axis) was independent of the flow rate, being 1·00, 0·83, 0·73, and 0·38 at the above distances from the axis. The ratios are plotted against the distance from the axis in Fig. 4, and on the same figure is drawn the Bessel function of zero order (see " Theoretical Treatment ").

Theoretical Treatment.

In the above experiments air passed through a cylindrical column of charcoal. The walls of the tube, and therefore the edge of the column, were kept at a steady temperature. Let distances down the column be measured as x, and distances from the axis as r. Let the heater supplying heat to the column be at $x = 0$. We will examine the conditions in a ring of charcoal of radius r, thickness dr, and depth dx at a distance x from the heater. In the following treatment the expansion of the air flowing through the bed has been ignored. Since the absolute temperature of the air never increased by more than about 15% this omission is probably justifiable.

The heat gained and lost by the ring of charcoal may be represented as made up of three parts:

(i) By conduction horizontally; rate of loss of heat is

$$-k \,.\, 2\pi r \,.\, \mathrm{d}x \,\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T_c}{\mathrm{d}r}\right)\mathrm{d}r$$

where k is the thermal conductivity of the granular bed of charcoal and T_c is the charcoal temperature, measured from the wall temperature which is taken as our zero.

(ii) By conduction vertically; rate of loss of heat is

$$-k \,.\, 2\pi r \,.\, \mathrm{d}r \,\frac{\mathrm{d}^2T_c}{\mathrm{d}x^2}\,\mathrm{d}x$$

(iii) By transfer to the air; rate of loss of heat is

$$K \,.\, 2\pi r \,.\, \mathrm{d}r \,.\, \mathrm{d}x(T_c - T_g)$$

where T_g is the air temperature measured from the same zero as T_c. This assumes, as Schumann (*J. Franklin Inst.*, 1929, **208**, 405) and other workers (Walker, Lewis, McAdams, and Gilliland, " Principles of Chemical Engineering ", McGraw-Hill, p. 107) have done, that the transfer of heat from a solid to a gas in contact with it is proportional to the temperature difference between them; K is the constant for this heat interchange and is expressed in calories per unit volume per unit temperature difference.

When the steady state has been reached, the rate at which heat is lost must equal the rate at which it is gained, so we may write:

$$K(T_c - T_g) = k\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T_c}{\mathrm{d}r}\right) + \frac{\mathrm{d}^2T_c}{\mathrm{d}x^2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In passing through the above ring of charcoal the air is raised in temperature by dT_g and this requires a supply of heat of

$$\mathrm{d}T_g \,.\, S \,.\, 2\pi r \,.\, \mathrm{d}r \,.\, L\rho$$

where S and ρ are respectively the specific heat and density of the gas, and L is the linear flow rate in the free tube before the stream reaches the charcoal bed. The linear flow rate in the bed is L divided by the fractional free space in the bed. The heat to raise the temperature of the air is supplied in two ways:

(*a*) From the charcoal:

$$K \,.\, 2\pi r \,.\, \mathrm{d}r \,.\, \mathrm{d}x(T_c - T_g)$$

(*b*) Because of cross channels between the granules, parts of the air stream will, at times, move laterally while flowing through the bed. This will result in a horizontal transfer of heat by convection, which will be given by a term of the form

$$C \,.\, 2\pi r \,.\, \mathrm{d}x \,\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}T_g}{\mathrm{d}r}\right)\mathrm{d}r$$

For the steady state, since there is no change with time, we have

$$\frac{\mathrm{d}T_g}{\mathrm{d}x} = \frac{K}{\rho SL}\,(T_c - T_g) + \frac{C}{\rho SL}\,\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}T_g}{\mathrm{d}r}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let $\Delta T = T_c - T_g$; then equation (2) can be written as

$$\frac{\mathrm{d}T_c}{\mathrm{d}x} - \frac{\mathrm{d}(\Delta T)}{\mathrm{d}x} - \frac{K}{\rho SL}\,(\Delta T) - \frac{C}{\rho SL}\,\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}T_c}{\mathrm{d}r}\right) + \frac{C}{\rho SL}\,\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}(\Delta T)}{\mathrm{d}r}\right) = 0$$

Substitution in this of (ΔT) from equation (1) gives

$$\frac{\mathrm{d}^3T_c}{\mathrm{d}x^3} + \frac{K}{\rho SL}\cdot\frac{\mathrm{d}^2T_c}{\mathrm{d}x^2} - \frac{K}{k}\cdot\frac{\mathrm{d}T_c}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x}\cdot\frac{1}{r}\cdot\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}T_c}{\mathrm{d}r}\right) + \frac{K}{\rho SL}\cdot\frac{1}{r}\cdot\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}T_c}{\mathrm{d}r}\right)$$
$$+ \frac{C}{\rho SL}\,\frac{K}{k}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}T_c}{\mathrm{d}r}\right) - \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}(\Delta T)}{\mathrm{d}r}\right)\right] = 0 \quad . \quad . \quad (3)$$

It will be seen later that, to explain the experimental results, K has to be assumed to be large; *i.e.*, the air temperature follows the charcoal temperature very closely through the column and (ΔT) is always very small. Therefore the last term in equation (3) may be neglected. This has been tested and found to be justified.

Let it be supposed that, to a first approximation, we may represent $T_c(x, r)$ as the product of a part dependent on x only and a part dependent on r only; *i.e.*,

$$T_c(x, r) = T_c{}^*(x) \,.\, \mathrm{f}(r) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is supposed that $T_c{}^*$ is the temperature of the charcoal on the axis of the tube and that $\mathrm{f}(r)$ varies from unity at $r = 0$ to zero at the outside edge of the charcoal ($r = R$). Substitution of (4) into (3) gives

$$\frac{\mathrm{d}^3T_c{}^*}{\mathrm{d}x^3} + \frac{K}{\rho SL}\,\frac{\mathrm{d}^2T_c{}^*}{\mathrm{d}x^2} - \left[\frac{K}{k} - \frac{\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\,(r\mathrm{f}'(r))}{\mathrm{f}(r)}\right]\frac{\mathrm{d}T_c{}^*}{\mathrm{d}x} + \left[\frac{C+k}{k}\right]\frac{K}{\rho SL}\cdot\frac{\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\,(r\mathrm{f}'(r))}{\mathrm{f}(r)}\,T_c{}^* = 0 \quad . \quad (5)$$

For equation (5) to hold and (4) to be true we must have :

$$\frac{\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\,(r\mathrm{f}'(r))}{\mathrm{f}(r)} = \text{a constant } (A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The solution of (6) which satisfies the boundary conditions, $\mathrm{f}(0) = 1$ and $\mathrm{f}(R) = 0$, is the Bessel function of zero order, $\mathrm{J}_0(y)$, where $y = -\sqrt{A}\,.\,r$. Since $\mathrm{J}_0(y) = 0$ when $y = 2{\cdot}405$,

$$A = -\,(2{\cdot}405/R)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This can be substituted back into (5) to give the following equation :

$$\frac{\mathrm{d}^3T_c{}^*}{\mathrm{d}x^3} + \frac{K}{\rho SL}\cdot\frac{\mathrm{d}^2T_c{}^*}{\mathrm{d}x^2} - \left[\frac{K}{k} + \left(\frac{2{\cdot}405}{R}\right)^2\right]\frac{\mathrm{d}T_c{}^*}{\mathrm{d}x} - \left[\frac{C+k}{k}\right]\frac{K}{\rho SL}\left(\frac{2{\cdot}405}{R}\right)^2 T_c{}^* = 0 \quad . \quad (5a)$$

The general solution of this equation is

$$T_c{}^*(x) = \Sigma a e^{mx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where the values of m are solutions of the cubic equation

$$m^3 + \frac{K}{\rho SL}\,m^2 - \left[\frac{K}{k} + \left(\frac{2{\cdot}405}{R}\right)^2\right]m - \left[\frac{C+k}{k}\right]\frac{K}{\rho SL}\left(\frac{2{\cdot}405}{R}\right)^2 = 0 \quad . \quad . \quad (9)$$

It is found that there is one positive root (m_1) and two negative roots (m_2 and m_3). Since T_c* must become zero when x goes to either plus or minus infinity, we must have:

For $x < 0$ $\quad T_c^* = a_1 e^{m_1 x}$

For $x > 0$ $\quad T_c^* = a_2 e^{m_2 x} + a_3 e^{m_3 x}$

Substituting in equation (1), we obtain:

For $x < 0$ $\quad \Delta T^* = b_1 e^{m_1 x}$

For $x > 0$ $\quad \Delta T^* = b_2 e^{m_2 x} + b_3 e^{m_3 x}$

The b values can be calculated from the a and m values by using equation (1); a_2 and a_3 can be calculated in terms of a_1 by using the condition that at $x = 0$, T_c* and (ΔT*) must be single valued. That is, the equations that give their values on each side of zero must lead, at zero, to the same value; a_1 can then be fixed to give the curve appropriate to the particular heating current used.

Numerical Calculations.—In Fig. 4 the experimentally determined temperature distribution across the tube was compared with the appropriate Bessel function distribution. Because the distances involved were small (four thermocouples in a distance of 2 cm.) we consider that the experimentally determined distribution across the tube was as close as could be expected to that deduced theoretically from equation (7).

The experiments were carried out in a tube having $R = 2{\cdot}35$ cm., which gives $A = -1{\cdot}047S\rho$; the number of calories required to heat 1 c.c. of gas through 1° at constant pressure is taken as $3{\cdot}125 \times 10^{-4}$. The relation between L, the linear flow rate, and F, the volume flow rate, is $1041L = F$, if F is expressed in c.c./min. and L in cm. per second. For example, $F = 5000$ c.c./min. corresponds to $L = 4{\cdot}804$ cm./sec. The values of k and K are not known for the particular bed of charcoal used, so they are taken to give the best fit to the experimental results. The values used are:

$k = 3{\cdot}2 \times 10^{-4}$ cal. per sq. cm. per sec. per unit temperature gradient;

$K = 2{\cdot}1 \times 10^{-2}$ cal. per c.c. per sec. per unit temperature difference.

The value of C, over the range of flow rates used, was obtained from the purely empirical formula

$$C \times 10^4 = 0{\cdot}23(F/1000) - 0{\cdot}325 = 0{\cdot}24L - 0{\cdot}325$$

This gives values of C increasing as the flow rate increases. This would be expected, since movement in the cross channels would be expected to increase with increasing flow rate. This will make the lateral transfer of heat greater.

The values of m for the different flow rates are obtained by solving the cubic equation (9). The values of a_2/a_1 and a_3/a_1 are obtained by applying the boundary condition that T_c * and (ΔT *) shall be single valued at $x = 0$. Equation (1) then gives values of b_1, b_2, and b_3 relative to a_1. The results are shown in Table VII.

TABLE VII.

F.	$C \times 10^4$.	m_1.	m_2.	m_3.	a_2/a_1.	a_3/a_1.	b_1/a_1.	b_3/a_1.
2000	0·135	2·244	−36·74	−0·4618	0·0036	0·9964	0·0608	−0·0127
2500	0·250	2·582	−30·15	−0·4056	0·0072	0·9928	0·0856	−0·0134
3000	0·365	2·901	−25·85	−0·3627	0·0124	0·9876	0·1123	−0·0138
3500	0·480	3·201	−22·85	−0·3291	0·0194	0·9806	0·1402	−0·0140
4000	0·595	3·478	−20·67	−0·3023	0·0281	0·9719	0·1683	−0·0142
4500	0·710	3·737	−19·00	−0·2803	0·0385	0·9615	0·1967	−0·0142
5000	0·825	3·970	−17·69	−0·2621	0·0502	0·9498	0·2384	−0·0142

The theory predicts that the temperature falls off exponentially on the near side of the heater ($x < 0$). For $x > 0$, the temperature is the sum of two exponential terms; but, since m_2 has a large negative value and a_2 is small, this term is never big and becomes quite negligible when x is still quite small. So, to a very close approximation:

For $x >$ *ca.* 0·1 cm., $\quad T_c^* = a_3 e^{m_3 x}$

Therefore our theory predicts that the temperature falls off exponentially on the far side of the heater, except when x is very small. The value of m_3, for a given charcoal bed, depends only on the flow rate and is predicted to be independent of the rate of heat supply. This was found experimentally (see Table IV and Fig. 3).

In the experimental section the results were shown graphically in the form :

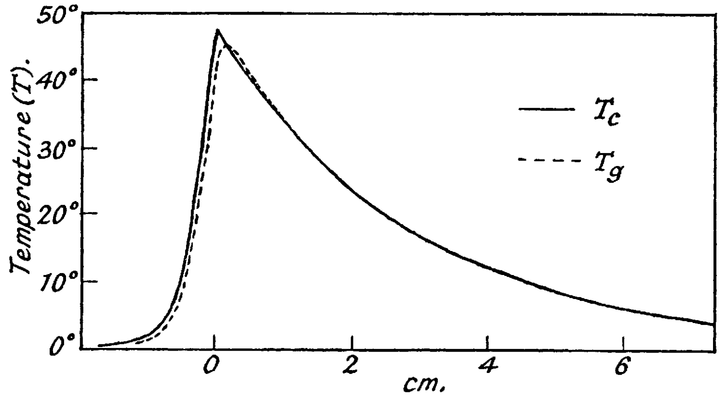
$$\text{For } x < 0, \qquad \log_{10}T = X + m_1'x$$
$$\text{For } x > ca.\ 0{\cdot}1 \text{ cm.}, \qquad \log_{10}T = Y + m_3'x$$

X and Y were assumed identical. On the present theory, X and Y differ slightly (always less than 2%). The values of m_1' and m_3' obtained theoretically (from m_1 and m_3 in Table VII) are compared in Table VIII with the experimental values. The value of Y is chosen to give the best fit with the experimental results, and from this and a_2/a_1, $(Y - X)$ is calculated. The calculated and experimental m' values are seen to agree well. In Fig. 2 the agreement is shown

TABLE VIII.

Flow rate in c.c./min.	m_1'. calc.	m_1'. obs.	m_3'. calc.	m_3'. obs.	Y.	$(Y - X)$.
2000	0·974	0·950	−0·2005	−0·1948	1·364	0·0015
2500	1·121	1·101	−0·1762	−0·1768	1·337	0·0032
3000	1·260	1·202	−0·1575	−0·1584	1·304	0·0054
3500	1·390	1·387	−0·1429	−0·1444	1·289	0·0085
4000	1·510	1·474	−0·1313	−0·1314	1·264	0·0124
4500	1·623	1·620	−0·1218	−0·1218	1·233	0·0171
5000	1·724	1·718	−0·1138	−0·1128	1·209	0·0223

FIG. 5.



graphically where, for $F = 2000$ and 5000 c.c./min., the graph is that theoretically deduced. The points are experimental.

The above theory was developed for an infinite layer, whereas the experiments were carried out on a finite layer. However, because the temperature falls off rapidly on the near side of the heater, the temperature at the top of the column is nearly equal to that of the wall and incoming gas ($T = 0$), so very little difference may be expected between the behaviour of the finite and an infinite column.

Discussion.

The agreement between theory and experiment shown in Table VIII suggests that the theory gives an accurate picture of the thermal conditions in the charcoal bed. It is important to compare the present theory with others. One important assumption made here was that the heat interchange between gas and solid was proportional to the temperature difference between them. The results suggest that this is valid. It was assumed by Schumann in his treatment, and Furnas's results supported Schumann's assumption. Also, results for heat transfer between the walls of a tube and the gas flowing through it have been satisfactorily explained by a similar assumption.

The value of the constant K, for this heat interchange, was required to be 0·023. Furnas gives values of this constant as it depends on granule size. Because his granules were larger than ours he obtained, in the main, rather smaller values of K, namely, *ca.* 0·01. However, the two values are of the same order of magnitude. Moreover, the value we used is very much what would be expected if Furnas's results are extrapolated to granules of the size we used.

The value assumed for k, the thermal conductivity, was 0·00032. This is of the correct order of magnitude for the thermal conductivity of such a bed of granules (see, *e.g.*, the work of Ingersoll and Koepp, *Physical Rev.*, 1924, **24**, 92). In the International Critical Tables the thermal conductivity of coke dust is given as 0·0004 and that of charcoal as 0·00014. So the value we found it necessary to assume is reasonable.

The fact that K, the constant for heat interchange, is much greater than k, the thermal conductivity, means that, when the steady state is reached, the air temperature is very close to the charcoal temperature throughout the column. In Fig. 5, T_c^* and T_g^* down the axis of the charcoal are plotted for $F = 3500$ c.c./min. The difference between them is small at all points.

Previous workers studying the interchange of heat between a solid and a gas flowing over it have considered that the rate of heat interchange depends on the gas velocity (Walker, Lewis, McAdams, and Gilliland, *op. cit.*, p. 109). We have taken K to be independent of the flow rate. This distinguishes our treatment from those of other workers. However, we have taken lateral movement of air in the bed into account (the term involving C). It has been necessary to make the reasonable supposition that C is dependent on the flow rate. So the condition of heat passage to and fro does depend on the flow rate. If the term involving C had been omitted from the theoretical treatment, it would have been necessary to assume that K was dependent on the flow rate. However, our results have been successfully accounted for by the much more reasonable assumption of a constant K and a lateral movement of air within the bed, which must certainly take place; this lateral movement depending on the flow rate.

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INORGANIC CHEMISTRY LABORATORY, OXFORD. [*Received, July 9th*, 1946.]